Qualifying Exam Critical Review

N. Chancellor

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Summary: Quantum Annealing With Manufactured Spins[1]

- Published in 2011
- Experimental paper attempting to demonstrate that in certain temperature regimes quantum annealing effects dominate over thermal annealing effects on D-wave artificial spin hardware
- Evidence based on temperature independence of freeze out time (details to be given later)
- Both single and multi q-bit experiments
- Numerical simulations compared with actual data

Quantum Versus Thermal Annealing

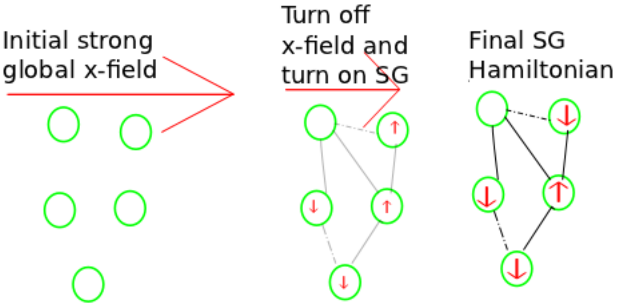
1. Quantum Annealing[1]: System is initially prepared in an easily prepared ground state[2] (maps to solution to an easy problem) and Hamiltonian is slowly changed to one where the ground-state maps to the solution of a non-trivial problem
   - Usually performed at relatively constant low temperature[3]
   - Time required for accurate result related to size of energy gap
   - Can be considered a form of quantum computing
2. Thermal Annealing: System is initially prepared at an effectively high temperature and then slowly cooled to low effective temperature
   - Effective change in temperature can be achieved by either lowering actual temp. or increasing Hamiltonian energy
   - Can be used for computation, but is not a form of quantum computing
   - Cooling too quickly results in system getting 'stuck' in a local minimum

Which is the D-wave hardware using?

- Non-trivial to test because "correct" solution (i.e. ground-state of final Hamiltonian) can be achieved either way
  1. System could remain mostly in the ground-state throughout process, thus achieving correct final ground-state (quantum annealing)
  2. During the evolution, noise or something else could put the system into a superposition of excited states (thermal fluctuations) and system could thermally cool until it is in the correct final ground state (thermal annealing)
- In principle quantum annealing should give a speedup, but with no useful theoretical bound on the speed of the classical algorithm [2](pg. 40) this cannot be used as a test
- Need another test for quantum behavior

The Annealing Protocol for Computation

- Initially Hamiltonian has no bonds or z-direction fields, but strong uniform x-direction field, simple ground-state
- Slowly x-direction field is turned off and bonds and z-direction fields are turned on
- Final Hamiltonian is known as an 'Ising spin glass', finding the ground-state of such a system is known to be NP-hard [3]



Freeze Time Experiments

1. System can be modeled as a double (or multiple) well
2. Reducing the x direction field can be though of as raising a barrier height between classical states in the z basis
3. t_{freeze}can be thought of as the time when the barrier becomes too high for tunneling

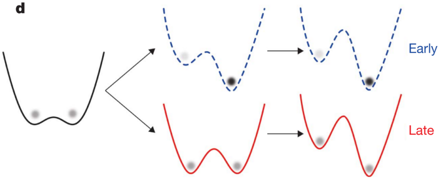
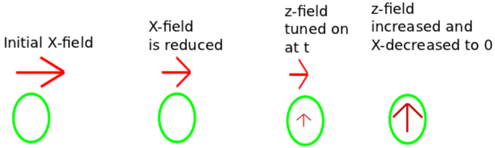


Figure: reprinted from [1]

Experiment I: Single q-bit

- Initially prepared in ground-state with field in x-direction
- x-direction field slowly turned off
- At a time t, a z-direction field is suddenly turned on
- Experiment is performed many times for a given range of values of t and probability of spin being 'up' is recorded
- Time when probability of being up becomes 50%[4] is called t_{freeze}because at this point the dynamics have been effectively frozen
- Experiment is performed at different temperatures



Results: Fidelity Versus Delay time

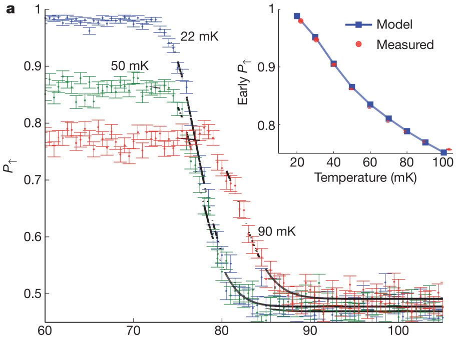


Figure: Probablity of being found in the ground state at the end of the annealing process versus t, the time at which the field on the intermediate spins is turned on for several different temperatures. Inset, probability of being in final ground state versus temperature, for an early t. Reprinted from [1]. Note that in this case the fidelity is very high at low temperature and the model fits very well for early times.

Results: t_{freeze}versus T

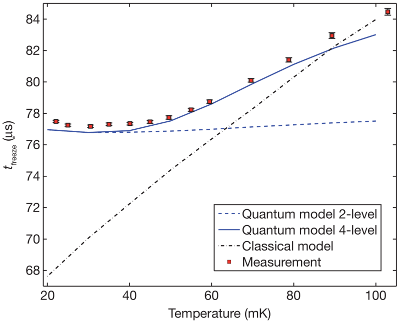


Figure: Actual measurement of t_{freeze}versus Temperature compared with classical and quantum models. Reprinted from [1] (error bars are 1 standard deviation). The classical model is a Monte Carlo analysis using the full Hamiltonian of each artificial q-bit (in this case there is only 1). The Quantum 2-level model uses Ising Hamiltonian as the full Hamiltonian of the system. The Quantum 4 level adds 1 ancilla coupled to each spin (again only 1 here), to represent internal degrees of freedom.

Interpretation

- Thermal annealing needs thermal fluctuations to reach ground state so therefore freeze time should decrease to zero as temperature decreases to zero
- Quantum annealing is not thermally driven and therefore should work all the way down to absolute zero
- Experimental data are consistent with a system which is dominated by thermal effects at higher temperatures and quantum effects at lower temperatures

Note However:

1. While building a classical (thermally driven model) which has a finite t_{freeze}at absolute zero is impossible, building one in which t_{freeze} plateaus for a certain range of temperature and than starts decreasing again should be possible[5], so the shape of the data alone is not conclusive evidence of quantum behavior
2. The best quantum model fits much better than the classical model but still doesn't match the data within error bars at most temperatures. The model doesn't match at low temperature, where keeping only a few levels in the model should be ok, therefore something may be missing from this model.
3. This experiment provides no evidence that tunnelling between q-bits, which is needed for meaningful calculations can be achieved (this is addressed later in the paper)

8 q-bit test

- Same experiment as before but with a ferromagnetic chain (coupling strength J) of 8 q-bits with strong fixed z-direction fields on either end in opposite directions (strength ± 2J)
- Weak (0.1J) z-direction field is placed on intermediate spins a time t
- For $t \gg t_{freeze}$ one would expect a probability of $\frac{1}{7}$ of finding spin in the lowest energy state ($|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle$) and equal probability of finding it in $|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle \ldots |\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$
- For $t \ll t_{freeze}$ one would expect the probabilities to no longer depend on t, and to be $> \frac{1}{7}$ to be in the state $|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle$
- A transition is expected for $t \sim t_{freeze}$

Graphic for 8 q-bit case

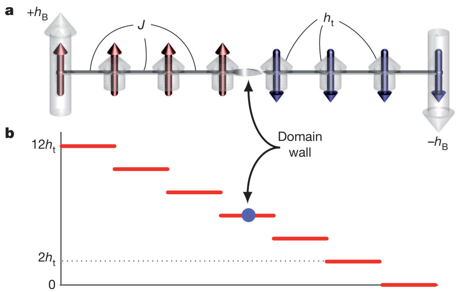


Figure: Graphical representation of the 8 q-bit system with energies of each of the 7 states mentioned on the previous slide, reprinted from [1]. Expressed as an equation the Hamiltonian is $H(t', t) =$
$A(t')\sum_{i=1}^{8}\sigma_i^x + B(t')J(2(\sigma_1^z - \sigma_8^z) + \sum_{i=1}^{7}\sigma_i^z\sigma_{i+1}^z + 0.1\Theta(t'-t)\sum_{i=2}^{7}\sigma_i^z)$

8 q-bit results I

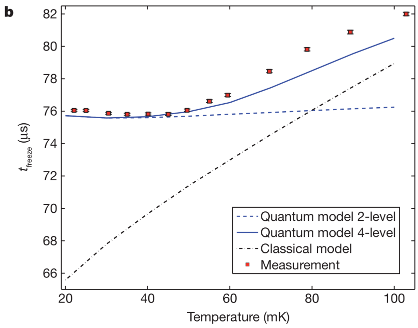


Figure: Actual measurement of t_{freeze}versus Temperature compared with classical and quantum models. Reprinted from [1] (error bars are 1 standard deviation). This plot is very similar to the single q-bit case, and the same conclusions can be drawn, but the same warnings also apply. Interestingly here t_{freeze} seems to slightly increase at lower temperatures, no possible explanation for this is given in the paper.

8 q-bits Results II

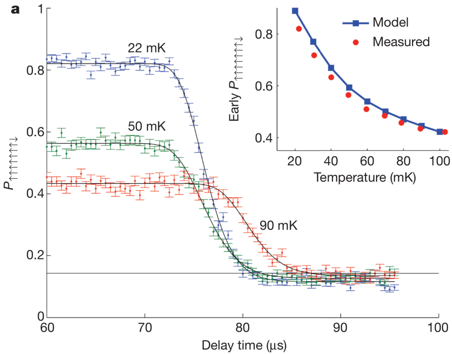


Figure: Probablity of being found in the ground state at the end of the annealing process versus t, the time at which the field on the intermediate spins is turned on for several different temperatures. Inset, probability of being in final ground state versus temperature, for an early t. Reprinted from [1].

Comments

- Note that even at the lowest temperature used and the earliest t the fidelity with the final ground state is only a bit more than 80%. This experiment wasn't performed very close to the adiabatic limit. It is probably reasonable to assume that the trends here would be the same for longer processes, but seeing direct evidence would have been better.
- The model fits fairly well, but seems to differ from the data at lower temperatures where the quantum dynamics dominate, this may indicate that some details of the quantum behaviors are missing from the model, perhaps with the couplers.
- Neither of these were an issue for the single spin case

Conclusions

1. This paper provides fairly convincing evidence that quantum behaviors dominate the processes in the artificial spin system at low temperatures.
2. This paper cannot be viewed as stand alone evidence that the D-wave hardware is a quantum computer, however it does provide compelling evidence that quantum dynamics are important in the processes which take place in the hardware.
3. Fundamentally the science of this paper is sound, but one should be careful to remember that, while scientifically important, the scope of these results are somewhat limited; this paper does demonstrate behavior which suggest possible use as a quantum computer, but does not provide a demonstration of quantum computing.
4. Even if quantum computing is demonstrated, a speedup is not guaranteed, again see [2](pg. 40).

Endnotes

[1] Some prefer to call this process "quantum adiabatic evolution", however I chose "quantum annealing" so that I could draw the parallel with thermal annealing

[2] Technically this can be done in any eigenstate, but the ground state is usually used for implementation reasons

[3] compared to the system gap

[4] Roughly speaking at least, this value is actually found with a complicated numerical fit, see [1]

[5] For example by having a large energy gap somewhere in the spectrum.

Citations

- [1] M. W. Johnson et al. Nature 473, 194-198 (12 May 2011)
- [2] Nielsen, Michael A. & Chuang, Isaac L. (2000), Quantum Computation and Quantum Information, Cambridge University Press
- [3] F. Barahona J. Phys. A: Math. Gen. 15 3241 (1982)

Background: the Hardware

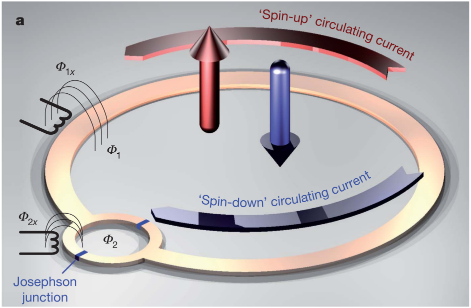


Figure: Schematic picture of a single D-wave superconducting flux q-bit reprinted from [1]. These q-bits are coupled together using electromagnetic couplers which provide a mutual inductance between ϕ_1 on two different q-bits bonds in an effective Ising model. Magnetic flux ϕ_1 can be though of as a z-direction magnetic field applied in an effective Ising model, where as ϕ_2 can be though of as an effective x- direction magnetic field.