Summary of current work

Jülich visit 2023

Nicholas Chancellor

March 30 2023

Engineering and Physical Sciences Research Council

Durham University

QCi Innovative Quantum Solutions Company

What this talk is about (+ collaborator acknowledgments)

A quick summary of some things I am working on, designed to stimulate discussion rather than give a full review

1. Why we need theory away from the adiabatic limit and a start on developing such theory
   - Work with Adam Callison, Max Festenstein, Jie Chen, Laurentiu Nita, and Viv Kendon

   Durham University

2. Encoding and hybrid algorithms, and future outlooks of these
   - Work with Adam Callison, Jie Chen, Tobias Stollenwerk, Puya Mirkarimi, Jesse Berwald (QCI) and Raouf Dridi (QCI)

   Durham University QCi Innovative Quantum Solutions Company

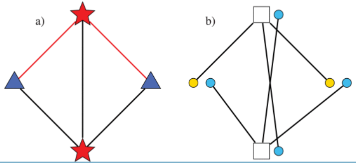
3. Use cases: how to actually apply quantum annealing (with example)
   - Work with Omer Rathore, Puya Mirkarimi, Alastair Basden, and Halim Kusumaatmaja

   Durham University

Past work worth mentioning: Coherent Pauli Checks (CPC)

A different perspective on (gate model) error correction/mitigation

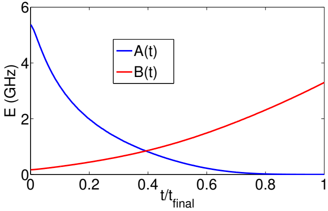
- Instead of considering abstract mathematical structures design codes based on actual parity checking operations
- Developed by myself and others and well suited to NISQ error mitigation, direct connection to classical codes ⋆

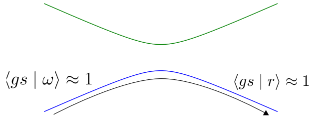


- Idea recently picked up by the IBM group: arχiv:2212.03937
- I'm not doing active research on this, but could be restarted
- May want to reach out to Joschka Roffe, wrote his thesis on them, currently in Jens Eisert's group at Free University Berlin

⋆Roffe...NC IEEE Transactions on Information Theory, 66 **1** 130 -146(2020); NC... arχiv:1611.08012; Roffe...NC... 2018 Quantum Sci. Technol. 3 035010

Advantages and disadvantages of adiabatic picture





$$H(t) = A(t)H_{\mathrm{driver}} + B(t)H_{\mathrm{problem}}$$

Theoretically satisfying

- Algorithm is effectively deterministic → "always" succeeds
- Intuitive picture involving only ground and first excited state

Let's assume P≠NP

- Algorithm succeeds roughly 100% of the time
- Total runtime needs to be exponential in size of problem → system needs to remain coherent for exponentially long time*

*There are ways to apply more sophisticated adiabatic theorem to faster quenches in some cases but that isn't the topic of this talk

Rapid quenches?

Energy conservation argument extended to any monotonic (closed system) quench

$$H(t) = A(t)\,H_{\mathrm{drive}} + B(t)\,H_{\mathrm{problem}} \quad \frac{A(t)}{B(t)} \geq \frac{A(t+\delta t)}{B(t+\delta t)} \forall_t$$

Sketch of proof:

1. Trotterize time evolution: $A(t) \rightarrow A(t+\delta t)$ and $B(t) \rightarrow B(t+\delta t)$ and apply $|\psi(t+\delta t)\rangle = \exp(-iH(t)\delta t)|\psi(t)\rangle$ in separate steps
2. Rescale time so that Hamiltonian always resembles (energy conserving) quantum walk $H_{eff}(\Gamma(t)) = \Gamma(t)\,H_{\mathrm{drive}} + H_{\mathrm{problem}}$
3. In rescaled version $\Gamma(t) \geq \Gamma(t+\delta t)$ (lowest $\langle H_{\mathrm{drive}}\rangle$ is $-n$) $\therefore$ $\langle H_{eff}(\Gamma(t))\rangle_{\psi(t)} - \Gamma(t)\,n \geq \langle H_{eff}(\Gamma(t+\delta t))\rangle_{\psi(t)} - \Gamma(t+\delta t)\,n$
4. Because $\langle H_{eff}(\Gamma(t))\rangle_{\psi(t)} \geq -\Gamma(t)\,n \ \forall_t$, $\langle H_{\mathrm{problem}}\rangle_{\psi(t)} \leq 0 \ \forall_t$

Details can be found in Callison et. al. PRX Quantum 2, 010338

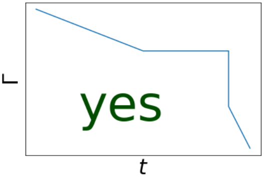
A very general result!

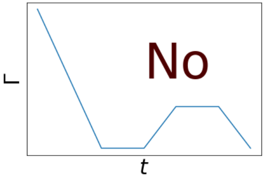
For result to hold (to be better than random guessing on average):

1. Monotonic $\Gamma(t) \geq \Gamma(t+\delta t)$ where $\Gamma(t) = \frac{A(t)}{B(t)}$
2. Start in ground state of H_{drive}
3. Driver not gapless $\rightarrow$ not a concern for real problems

What is allowed:

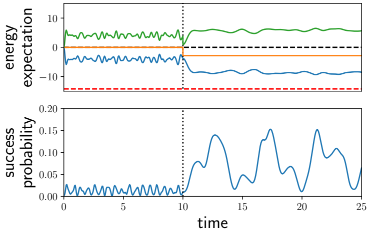
1. No limit on how fast algorithm runs
2. Discontinuities in $\Gamma(t)$ are ok
3. H_{drive} does not need to be diagonal in an orthogonal basis to H_{problem} $\rightarrow$ starting state can be biased

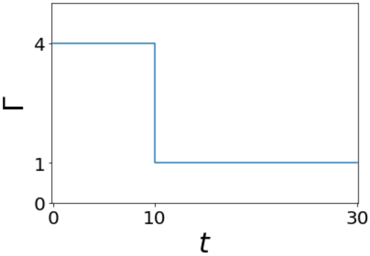




Intuitive example: two stage quantum walk

Perform a quantum walk at γ_1, and than use result as an input state for a second walk at $\gamma_2 < \gamma_1$





- Energy expectations: Green$= \gamma_{1,2}\langle H_{\mathrm{drive}}\rangle$; Blue$= \langle H_{\mathrm{problem}}\rangle$; Gold$= \gamma_{1,2}\langle H_d\rangle + \langle H_{\mathrm{problem}}\rangle$
- Total energy conserved except for at dashed line where γ decreases
- Non-instantaneous quench effectively infinite stage quantum walk

Why is the rapid quench result important?

General, but rather weak:

Any monotonic quench at least as good as measuring the initial state

1. Design protocols to maximize dynamics $\rightarrow$ don't need to worry about dynamics being counter-productive
2. A **biased** search can already start from a very good guess
3. Mechanism to understand dynamics very far from adiabatic limit

The effect of encoding: domain-wall encoding

Consider higher-than-binary dis-
crete problems; appear often in real world optimisation, for example:

- A truck can go down any of three roads...
- A tasks can be scheduled at any of five times...
- A component can be placed any of seven places on a chip...

- Define two index objects:

$$x_{i,\alpha} = \begin{cases} 1 & \text{variable } i \text{ takes value } \alpha \\ 0 & \text{otherwise} \end{cases}$$

- Discrete Quadratic models, (DQM), made from pairwise interactions of x terms:

$$H_{\mathrm{DQM}} = \sum_{i,j} \sum_{\alpha,\beta} D_{(i,j,\alpha,\beta)} x_{i,\alpha} x_{j,\beta}$$

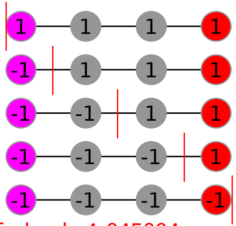
Discrete variables into binary, three ways

Variable size=m

performance metric	binary	one-hot	domain wall*
# binary variables	$\lceil \log_2(m) \rceil$	m	$m-1$
# couplers for encoding	0 if $m = 2^n$ $n \in \mathbb{Z}$ complicated otherwise	$m(m-1)$	$m-2$
intra-variable connectivity	N/A or complicated	complete	linear
maximum order needed for two variable interactions	$2\lceil \log_2(m) \rceil$	2	2

Binary= assign bitstrings to configurations
One hot= constrain variables so exactly one can be 1
Domain wall= new encoding w/ better performance†

encoded value	qubit configuration
0	1111
1	-1111
2	-1-111
3	-1-1-11
4	-1-1-1-1



*For details see: Chancellor, Quantum Sci. Technol. 4 045004
†Chen et. al. IEEE Transactions on Quantum Engineering 3102714 (2021)

Improved performance on maximum three colouring*

Green=statistically significant result (95% confidence)

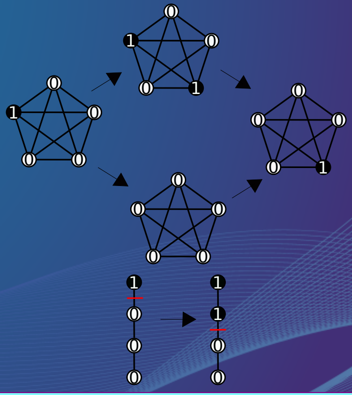
	Adv. dw/oh		2000Q dw/oh		dw Adv./2000Q		oh Adv./2000Q		(dw, Adv.)/(oh, 2000Q)		(dw, 2000Q)/(oh, Adv.)	
5 node (b,w)	0	0	0	0	0	0	0	0	0	0	0	0
5 node p												
10 node (b,w)	42	0	37	0	2	0	19	21	39	0	40	0
10 node p	2.27×10^{-13}		7.28×10^{-12}		2.50×10^{-1}		6.82×10^{-1}		1.82×10^{-12}		9.09×10^{-13}	
15 node (b,w)	85	2	95	3	32	34	70	22	94	1	91	2
15 node p	2.47×10^{-23}		4.95×10^{-25}		6.44×10^{-1}		2.67×10^{-7}		2.42×10^{-27}		4.41×10^{-25}	
20 node (b,w)	99	0	100	0	43	41	94	3	100	0	93	2
20 node p	1.58×10^{-30}		7.89×10^{-31}		4.57×10^{-1}		9.60×10^{-25}		7.89×10^{-31}		1.15×10^{-25}	
25 node (b,w)	100	0		FAIL	66	20		FAIL		FAIL	98	2
25 node p	7.89×10^{-31}				3.33×10^{-7}						3.98×10^{-27}	
30 node (b,w)	100	0		FAIL	72	20		FAIL		FAIL	97	2
30 node p	7.89×10^{-31}				2.30×10^{-8}						7.81×10^{-27}	
35 node (b,w)	100	0	FAIL	FAIL		FAIL		FAIL		FAIL	FAIL	
35 node p	7.89×10^{-31}											
40 node(b,w)	100	0	FAIL	FAIL		FAIL		FAIL		FAIL	FAIL	
40 node p	7.89×10^{-31}											

- Domain-wall on 2000Q beats one-hot on Advantage (100 total each size b=number better, w=number worse, p=statistical significance)
- Trend continue up to size where no longer possible to embed in 2000Q (FAIL), similar results for k-colouring (not shown)
- Worth trying if you have discrete problems to encode

*Chen et. al. IEEE Transactions on Quantum Engineering 3102714 (2021)

Effect on dynamics*

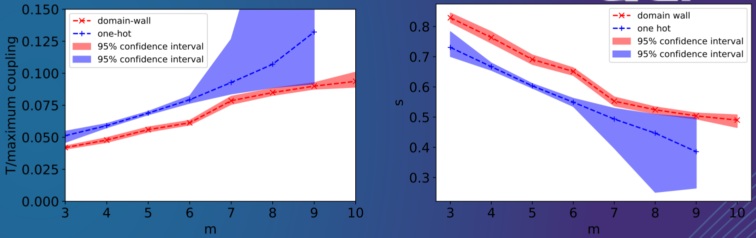
- One hot value cannot be changed by flipping a single binary variable
- Domain wall can therefore easier for transverse field to update



Need to consider underlying physics with encoding

*see: Berwald, Chancellor, Dridi arχiv:2108.12004

Effective temperature and freeze point*



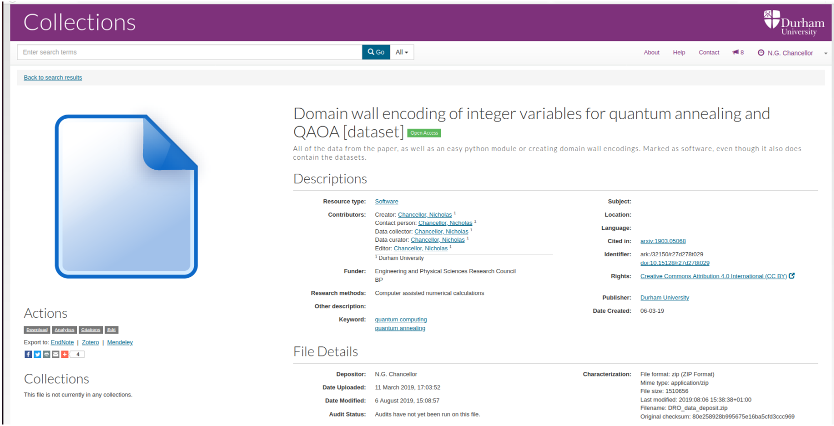
- Already taken into account embedding strength (see: paper for details)
- Domain-wall version effectively sampled at lower temperature $\leftrightarrow$ later freezing
- Work accepted (but not yet published) in Philosophical Transactions A

Encoding has a strong effect on the dynamics of how the problem is solved

*see: Berwald, Chancellor, Dridi arχiv:2108.12004

Want to try it yourself?

Python code to create domain wall encodings available at https://collections.durham.ac.uk/: “Domain wall encoding of integer variables for quantum annealing and QAOA [dataset]”*

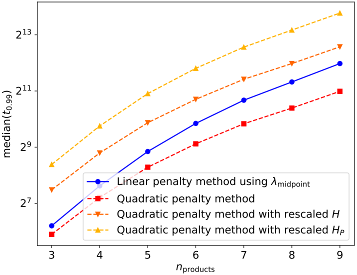


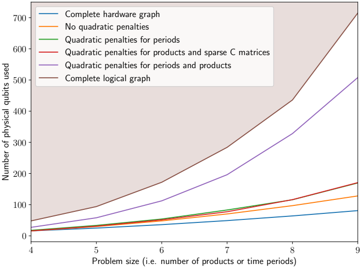
*https://doi.org/10.15128/r27d278t029

New work on encoding

Can we approximate k-hot constraints with linear terms only?

- Excellent idea by my student Puya Mirkarimi
- Work in progress, but promising initial results
- Work with dunnhumby, a consumer data science company, to examine realistic problems





Finding use cases for quantum computing

Very important to find good early use cases

- Quantum computers potentially very powerful in some ways, very limited in others
- In addition to finding the best algorithms, we need to find most promising use cases
- Not the subject of this talk, but some discussion on how to think about this systematically (in an industrial setting) in Nicholas Chancellor, Robert Cumming, Tim Thomas arχiv:2006.05846
- Working on projects to find these (see next slide)

Relevant UK projects for this audience

Collaborative computational project on quantum computing (CCP-QC)

- Work with other CCPs (academic projects) to find uses fro quantum computing within scientific research
- Idea is to use quantum computing to solve hard problems which come up in academic research rather than industry
- **https://ccp-qc.ac.uk/**

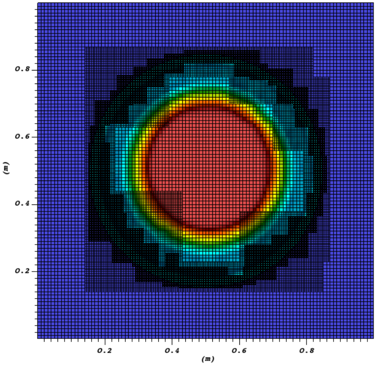
Quantum Enhanced and Verified Exascale Computing (QEVEC)

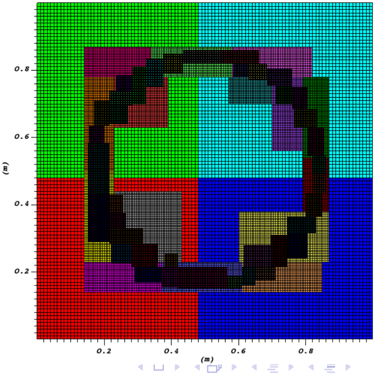
- Work on how quantum coprocessors can (eventually) support exascale computing
- Multiple projects looking at a variety of applications
- **https://excalibur.ac.uk/projects/qevec/**

Contact Viv Kendon at **viv.kendon@strath.ac.uk** if you are interested in potential collaborations

HPC load balancing, QEVEC project

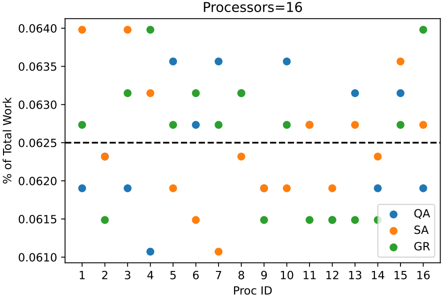
- Look at realistic cases, load balancing for fluids problems
- Some regions of space require a much finer mesh than others
- Given mesh, allocate points to cores so load is balanced (number partitioning)
- Example below, Sod shock left: density fields and mesh right: allocation of mesh points to cores





Some (very) preliminary results

- Start with simplest example (Sod shock)
- Iterative partitioning in 4 stages (only last shown), using simulated annealing (SA), quantum annealing (QA), and a greedy method (GR)



- Want as even as possible of a split
- Model is likely too simple, problem is "too easy" looking graph based partitioning