PhD. Thesis Defense

Nicholas Chancellor

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An Implementation of the Circuit Model of Quantum Computation by adiabatic transport: Motivation

- Currently, most popular implemented model of adiabatic quantum computation (Ising spin glass) is not universal[1], for instance it cannot efficiently implement Shors algorithm
- Multi-stage holonomic computation allows for arbitrary calculations to be performed even with limited connectivity
- Model only requires a moderate extension (ability to implement Heisenberg type qubits and bonding) of current capabilities of artificial spin qubit hardware [2, 3, 4, 5, 6]
- Other potential implementations of Heisenberg spin chain [7] using photon coupled micro cavities
- Can also be implemented non-adiabatically using the protocol from [8, 9, 10, 11]

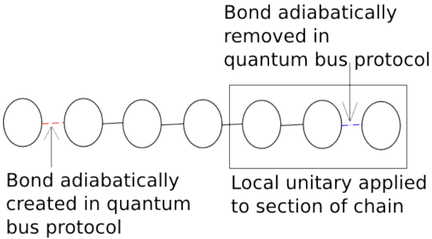
Presentation Structure

- Brief overview

1. Universal computation performed by twists
   1.1 single qubit gates
   1.2 CNOT gate
2. Adiabatic transport protocol
   2.1 overview
   2.2 results for transport protocol and 2 qubit gates
3. Implementation with superconducting flux qubits
   3.1 currently used Ising model
   3.2 proposed implementation for Heisenberg model

Overview I, Basic Idea:

- perform computation by using a transport protocol to transport a qubit state down a chain with a unitary 'twist' applied to it, also two qubit controlled gate extension



- can be thought of as a form of universal open loop holonomic quantum computation
- can be implemented using both adiabatic and non-adiabatic transport protocols

Overview II, What is Needed to be Universal?

A Universal set of quantum gates is a set of gates which can approximate up to arbitrary accuracy any unitary operation [12](pg. 188)

1. Universal set of logic gates [12](pg. 189):
   - Single qubit gates: Hadamard, phase[2], and $\frac{\pi}{8}$ or equivalent set of gates: combine to allow arbitrary rotations (at least to arbitrary precision) of any single spin
   - CNOT (controlled NOT) gate: performs a spin flip (or not) on a given spin based on the state of a single other spin
2. Way of getting qubits to gates (data bus), sufficient conditions:[3]
   - all single qubit gates can be performed on any qubit at any point in the computation
   - CNOT gate can be performed on any given pair of qubits at any time

Part 1.1: Single Qubit Gates

- Hamiltonian spectrum is unchanged by local basis rotations (or reflections) because these are unitary operations
- Any twist in the spin chain that changes $x, y, z \to x', y', z'$ for certain spins where x', y', z' are all mutually orthogonal to each other will not disrupt the spectrum and therefore will not affect dynamics
- twisted Hamiltonian:

$$\mathrm{H}_{\mathrm{twist}} = \sum_{i=1}^{N'-1} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + \vec{\sigma}_{N'} \cdot \vec{\sigma}'_{N'+1} + \sum_{j=N'+1}^{N-1} \vec{\sigma}'_j \cdot \vec{\sigma}'_{j+1}$$

$$\vec{\sigma}' = (\sigma^{x'}, \sigma^{y'}, \sigma^{z'})$$

- By performing a transport protocol (will return to the transport protocol later) using a twisted chain, any single spin operation can be performed

Example: Hadamard Gate

$$\mathcal{H}|\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} |\psi\rangle$$

Consider operation on Pauli matrix eigenvectors by $\mathcal{H}$:
$x_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow x'_+ = \mathcal{H}x_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = z_+$,$y_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \imath \end{pmatrix} \rightarrow$
$y'_+ = \mathcal{H}y_+ = \frac{1}{2} \begin{pmatrix} 1+\imath \\ \imath - 1 \end{pmatrix} = \frac{\imath - 1}{2} \begin{pmatrix} 1 \\ -\imath \end{pmatrix} = y_- \exp(\imath\phi)$,
$\phi \in \mathbb{R}$,$z_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow z'_+ = \mathcal{H}z_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = x_+$ Similarly for - eigenvectors
We can now deduce that to perform this twist $\sigma^x \rightarrow \sigma^z$, $\sigma^y \rightarrow -\sigma^y$and $\sigma^z \rightarrow \sigma^x$

Final Hamiltonian to Implement Hadamard

$$\mathrm{H}_{\mathrm{Hadamard}} = \sum_{i=1}^{N'-1} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + \sigma^x_{N'} \sigma^z_{N'+1} +$$

$$\sigma^z_{N'} \sigma^x_{N'+1} - \sigma^y_{N'} \sigma^y_{N'+1} + \sum_{j=N'+1}^{N-1} \vec{\sigma}_j \cdot \vec{\sigma}_{j+1}$$

Single Qubit gates and required twist

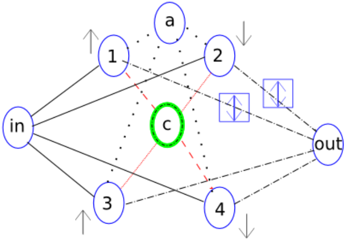
Gate Name	Matrix	$\sigma^{x'}$	$\sigma^{y'}$	$\sigma^{z'}$
Hadamard	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	σ^z	$-\sigma^y$	σ^x
$\frac{\pi}{8}$	$\begin{pmatrix} 1 & 0 \\ 0 & \exp(\imath\frac{\pi}{4}) \end{pmatrix}$	$\frac{1}{2}(\sigma^x + \sigma^y)$	$\frac{1}{2}(\sigma^y - \sigma^x)$	σ^z
phase	$\begin{pmatrix} 1 & 0 \\ 0 & \imath \end{pmatrix}$	$-\sigma^y$	σ^x	σ^z
NOT[4]	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	σ^x	$-\sigma^y$	$-\sigma^z$

Table: Required twist for given single qubit gates

Part 1.2 Controlled Gates

- To build a universal quantum computer we now only need a CNOT gate
- CNOT gate can be implemented by building a switch which chooses between two possible paths for the quantum bus, one which has a NOT twist and one which does not
- Strong local fields can effectively 'break'[5] a spin chain [14]
- An Ising type bond to a spin in a given direction, up or down can mimic a field in (opposing) the direction of the first spin for (anti-)ferromagnetic bonding
- Each 'active' spin should feel zero net bias toward being spin up or down from the fields

CNOT Implementation



= spin 1/2 control spin

= spin 1/2 working spin

=Initial AF Heisenberg bond (weak)

=Final AF Heisenberg bond (weak)

=Fixed AF Heisenberg bond (weak)

=Bond Twisted to perform 'Not' operation

=Fixed F Ising bond (strong)

=Fixed AF Ising bond (strong)

↑ =Fixed "up" magnetic field (strong)

↓ =Fixed "down" magnetic field (strong)

CNOT Operation

- Consider case where control spin is up:
  - Ising bonds exactly cancel fields in top channel on previous slide, and reinforce fields in lower channel
  - Strong fields effectively 'block' lower channel, one spin is pinned up and the other is pinned down; effects of bonds from this channel cancel completely by symmetry
- Same happens but with upper and lower switched if control spin is down
- Energy degeneracy guarantees that no phase difference will be accumulated between these states

CNOT example

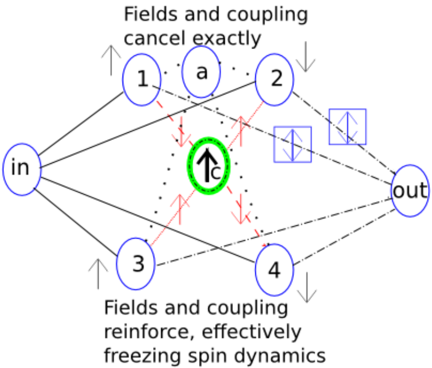


Figure: CNOT system with control spin up, will execute NOT twist on target spin under quantum bus protocol.

Part 2.1: Adiabatic Quantum Data Bus

- Consider Heisenberg Hamiltonian with odd chain length (N):

$$\mathrm{H} = \sum_{i=1}^{N-1} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} = \sum_{i=1}^{N-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z)$$

- Hamiltonian has block diagonal structure based on polarization sector i.e. all eigenstates can be expressed as states with a definite total polarization
- For anti-ferromagnetic case (the case given here) the ground state lies in the sector with minimum magnitude of total polarization
- Because of symmetry under spin flips, all eigenstates must be at least 2-fold degenerate (remember N is odd so there is no zero polarization sector).
- Twofold degeneracy can hold 1 qubit of information

Quantum Data Bus, Process

- Consider the following process:

1. An even length spin chain is prepared in its ground state (unique)
2. A single spin prepared in an arbitrary state $|\psi\rangle$ is slowly (slowly enough for the adiabatic theorem to apply) attached to one end of the chain, the system is now in a superposition state of the two states in the ground state manifold
    - degenerate energy prevents a phase difference from being acquired
    - block diagonal structure prevents exchange of amplitude between states
3. a single spin is now detached from the other end of the chain (slowly enough for the adiabatic theorem to apply), and the spin in an arbitrary state is transported from one end of the chain to the other up to U(1) phase factor

Quantum Data Bus, Cartoon

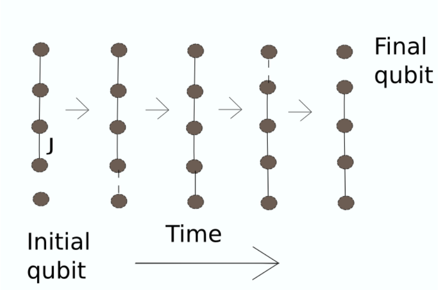


Figure: Cartoon of the quantum data bus protocol for a Heisenberg Hamiltonian.

Part 2.2: Transport Data Runs on the J1-J2 Heisenberg Spin Chain

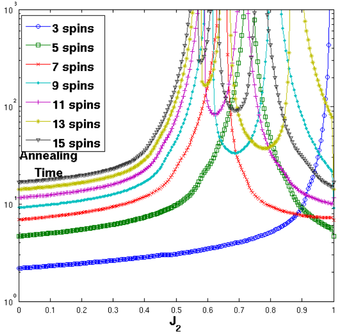
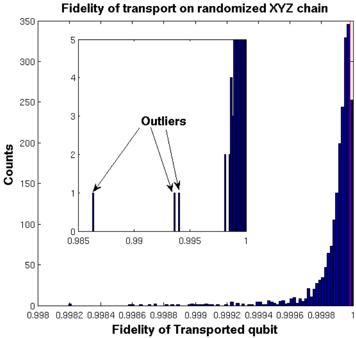


Figure: Plot of Annealing time in units of inverse Hamiltonian energy for various J_2 values (J_1 set to unity) versus chain length. This plot is time required to reach a 90% fidelity with the appropriate ground-state of the final Hamiltonian for the first part of the process shown in Fig. 2. Reprinted from [13].

Transport Protocol is not Ruined by Randomized Bond Strengths



Fid. for 7 spin XYZ chain with ±50% randomization annealed for t=10				
1-Mean	1-Median	1-clean	% above clean	# samples
$1.18 * 10^{-4}$	$6.42 * 10^{-5}$	$1.70 * 10^{-5}$	9.0%	2061

CNOT performance

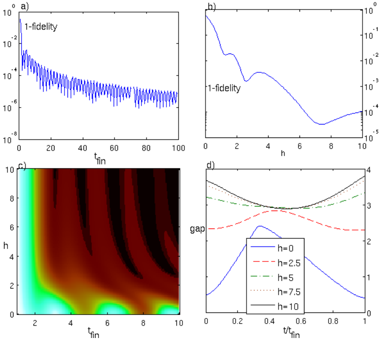


Figure: Measures of performance of the CNOT gate a) 1-fidelity of the output spin versus t_{fin} for h=10 b) 1-fidelity of the output spin versus h for t_{fin}=10 c) log of one minus output fidelity for initial up spin with a NOT performed versus h and t_{fin} light is larger (more positive), dark is smaller (more negative) d) gap versus $\frac{t}{t_{fin}}$ for various values of h.

Part 3.1: Current, non-Universal Flux Qubit Devices

- Would require modification of currently available hardware which implements an Ising spin glass [2, 3, 4, 5, 6]
- The effective single qubit Hamiltonian is [3]

$$\mathrm{H} = \sum_n \left(\frac{Q_n^2}{2C_n} + U_n \frac{(\phi_n - \phi_n^x)^2}{2}\right) - U_q \beta_{eff} \cos(\phi_q - \phi_q^0)$$

- based on so called compound compound Josephson junction (ccjj), see next slide $n \in \{q, cjj, l, r\}$

Ising Implementation

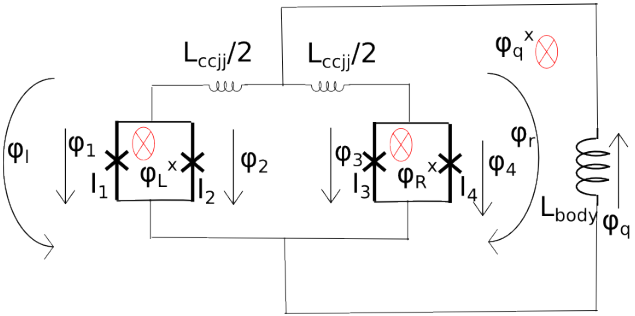


Figure: Currently implemented ccjj devices used for AQC, for examples of work on these devices, see [2, 3, 4, 5, 6]. Fluxes in the smallest pair of loops are used only to correct for lack of precision in Josephson junction fabrication.

Part 3.2: Proposal for a Heisenberg qubit

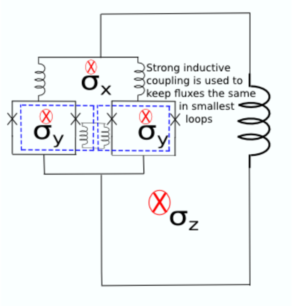


Figure: Proposed design for single Heisenberg qubit using a ccjj, for this design all Josephson junctions are assumed to be exactly identical. Application of fluxes is equivalent to adding spin operators to the Hamiltonian.

Design Analysis

- Requires inductive coupling between smallest loops.
- Manufacturing variance can be compensated for by building a compound compound compound Josephson junction (cccjj).
- Effective Hamiltonian is:

$$\mathrm{H} = \sum_n \left(\frac{Q_n^2}{2C_n} + U_n \frac{(\phi_n - \phi_n^x)^2}{2}\right) - U_q \frac{8\pi L_q I_c}{\Phi_0} \cos\left(\frac{\phi_y}{2}\right) \cos\left(\frac{\phi_{ccjj}}{2}\right) \cos(\phi_q)$$

- Simulates fields in all 3 directions, coupling can be achieved by inductively coupling loops of different qubits.

Conclusions

1. Open loop holonomic computing architecture based on twisted spin chains
   - Gates performed by transport protocol on chains or clusters of spins
   - Can be implemented either adiabatically using the transport protocol from [13] or non-adiabtatically using the protocol from [8, 9, 10, 11]
2. Specific application for holonomic computing using superconducting flux qubits
   - Strong experimental evidence in favor of non-Universal Ising spin glass implementation of AQC
   - Can implement universal HQC architecture with modified qubit circuits and coupling between all three effective spin directions

Endnotes

[1]See[12](188, 189 and 281) for criteria to be universal and compare to the capabilities listed in [2, 3, 4, 5, 6].

[2]this is not strictly necessary, but see [12]

[3]The conditions below are sufficient, but may not be necessary, that question is beyond the scope of this presentation

[4]This gate is needed for the construction of the CNOT

[5]By 'break' I mean that there exists a way of splitting any degeneracies where all eigenstates of a given spin chain can be written as states which have no entanglement between two bi-partitions.

Citations

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