

Continuous time quantum computing beyond adiabatic: quantum walks and fast quenches

Thematic program on quantum simulation

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A brief note about terminology

For the purposes of this talk:

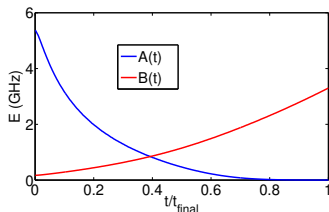
- ▶ **Adiabatic quantum computation (AQC)** → *closed system* protocols where an eigenstate is maintained via the adiabatic theorem of quantum mechanics
- ▶ **Quantum Annealing (QA)** → *dissipation* from open system effects is the dominant mechanism

The terminology is not standardized and different groups may use these terms differently

Adiabatic quantum computing

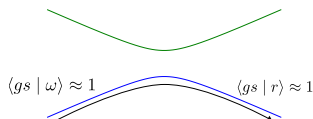
Traditional picture:

- ▶ Map an NP-hard optimization problem to a Hamiltonian, unknown ground state is solution
- ▶ *Slowly* change from a (driver) Hamiltonian with an easily prepared ground state to problem Hamiltonian
- ▶ Adiabatic theorem of quantum mechanics \rightarrow success probability arbitrarily close to 100 % by running long enough



$$H(t) = A(t)H_{\text{driver}} + B(t)H_{\text{problem}}$$

Advantages and disadvantages of this picture



Theoretically satisfying

- Algorithm is effectively deterministic \rightarrow “always” succeeds
- Intuitive picture involving only ground and first excited state

Let's assume $P \neq NP$ *

- Algorithm succeeds roughly 100% of the time
- Total runtime needs to be exponential in size of problem \rightarrow system needs to remain coherent for exponentially long time

*For those unfamiliar with complexity theory, this is basically saying “let's assume that hard optimization problems exist”, most experts believe $P \neq NP$

What can be done?

Restore coherence somehow

- Error correction, difficult to do in continuous time, but progress being made
- Low temperature dissipation can restore coherence \rightarrow would have to be very low temperature

- Have to mitigate *all* errors for a *very* long time
- Not the subject of this talk

Succeed with low probability

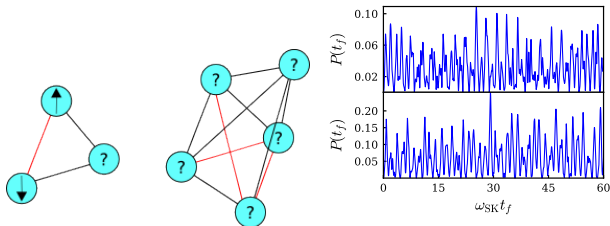
- *Total* runtime is still exponential in problem size
 - Each run is short \rightarrow exponentially many needed to hit right answer
-
- Exponentially low success each run is *conceptually* unsatisfying...
 - ... but much less demanding for coherence



Lottery

Example: continuous time quantum walk on spin glass

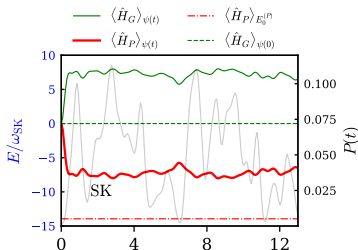
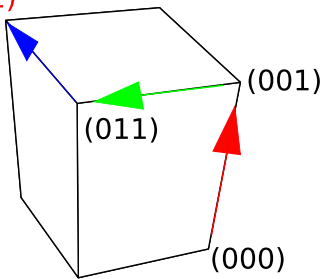
- ▶ Start with an equal positive superposition of all solutions, $|\omega\rangle = \frac{1}{\sqrt{N}} \sum_i |i\rangle$
- ▶ Evolve with a fixed Hamiltonian $H_{\text{walk}} = \gamma H_{\text{hop}} + H_{\text{problem}}$
- ▶ $H_{\text{hop}} = -\sum_i \sigma_i^x \rightarrow$ superposition is ground state
- ▶ $H_{\text{problem}} = \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z + \sum_i h_i \sigma_i^z$ where h_i and $J_{i,j}$ drawn from the same Gaussian distribution
- ▶ Measure after random short period of time, repeat many times



See [arXiv:1903.05003](https://arxiv.org/abs/1903.05003) for details, work with Adam Callison, Viv Kendon, and Florian Mintert

How is this a 'walk'? How does it find solutions?

(111)



- ▶ H_{hop} effectively forms a hypercube with a bitstring at each vertex, probability amplitude 'walks' between different states
- ▶ H_{problem} contributes phases which guide the walk

Energy is conserved $\langle H_{\text{walk}} \rangle_{t=0} = \langle H_{\text{walk}} \rangle_{t>0}$ since the system starts in the ground state of H_{hop} :

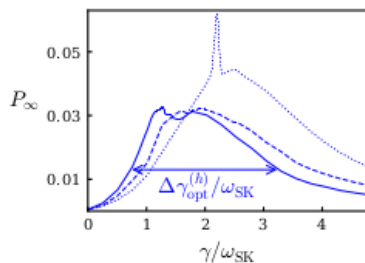
$$\langle H_{\text{problem}} \rangle_{t>0} - \langle H_{\text{problem}} \rangle_{t=0} = \langle H_{\text{hop}} \rangle_{t=0} - \langle H_{\text{hop}} \rangle_{t>0} \leq 0$$

Walk seeks out 'good' solutions!

How much to walk? Choosing the γ parameter

$$H_{\text{walk}} = \gamma H_{\text{hop}} + H_{\text{problem}}$$

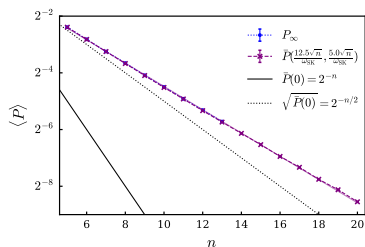
Still have one undefined parameter, γ , how do we set it? does it need to be set precisely? How do we make sure we are not 'cheating'?



P_{∞} is the long time average success probability

Short answer: yes, γ does not have to be precisely set to find solutions effectively, and we can find a heuristic to choose it (without cheating). For details about the heuristic see [arXiv:1903.05003](https://arxiv.org/abs/1903.05003)

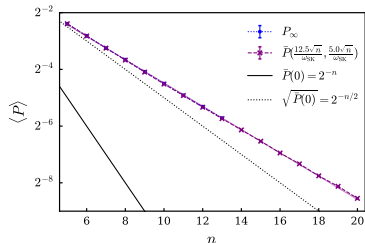
How well this works, numerically extracting scaling



- ▶ $N = 2^n$ possible bitstrings, one correct solution, runtime scales as inverse probability
- ▶ Scaling of $\frac{1}{N^{0.417}}$ better than both classical guessing ($\frac{1}{N}$) and $\frac{1}{\sqrt{N}}$ unstructured (Grover like) quantum search

The structure of the problem (correlations in bitstring energies) is playing a role in the computational mechanism, otherwise could not beat $\frac{1}{\sqrt{N}}$ scaling

A note on runtimes of single walk



There are actually two curves on top of each other in the above plot:

1. Infinite time average
2. Average over a time $\propto \sqrt{n}$ *

\therefore The system is equilibrating quickly

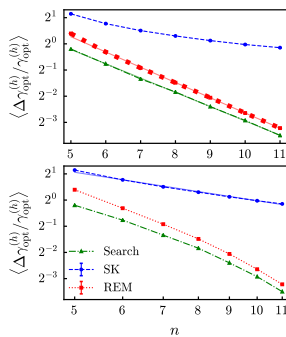
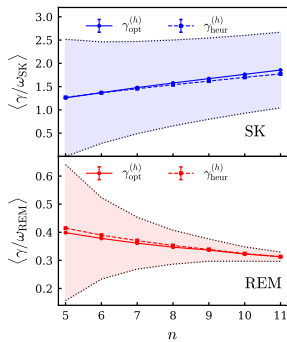
Implies that the walk is probably occurring in the precursor to a paramagnetic phase, rather than a spin glass phase \rightarrow spin glass phase would equilibrate slowly

*yes n , the number of qubits *not* $N = 2^n$

Compare to problems without correlations

Random Energy Model (REM): each bitstring is assigned a random independent energy

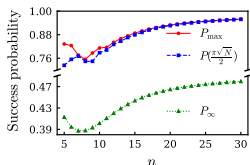
- ▶ Dynamics become dominated by a single close avoided crossing, require fine tuned γ , technically difficult, may not be possible to find correct value
- ▶ Requires single long run for high success probability \rightarrow need long coherence time



Compare to unstructured search

- ▶ Continuous time analog to Grover search
- ▶ Problem Hamiltonian is a single marked bitstring $|m\rangle\langle m|$:

$$H(s) = -(1-s) \sum_i \sigma_i^x + s(1 - |m\rangle\langle m|)$$

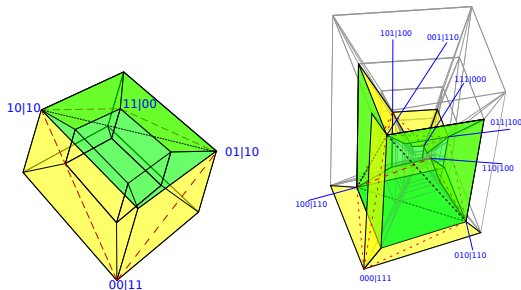


- ▶ Same optimal speedup as gate model finds solution in time \sqrt{N} rather than N from classical guessing/exhaustive search
- ▶ Exponentially sensitive to parameter setting (values of s)
- ▶ Succeeds with $O(1)$ probability after \sqrt{N} runtime

For a detailed study of AQC and QW, see [Morley et. al. Phys. Rev. A 99, 022339 \(2019\)](#), for practical implementation, see [Dodds et. al. Phys. Rev. A 100, 032320 \(2019\)](#)

Aside: Implementing unstructured search dynamics *

- ▶ Naively requires implementation of all 2^n Pauli strings \rightarrow $-|0\rangle\langle 0| = \prod_i \frac{1}{2}(1 - \sigma_i^z)$ not practical
- ▶ However auxiliary qubits can be used to 'count', spins ones in a configuration \rightarrow Apply penalty to counting spins and implement at second order of perturbation theory



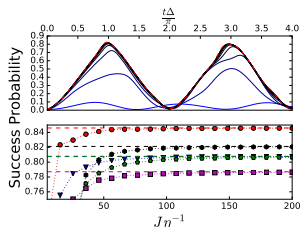
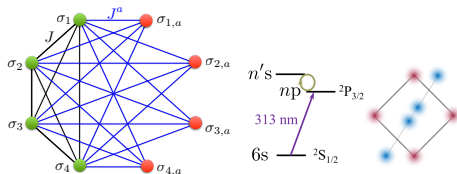
Embeds a hypercube in a hypercube of twice the dimension [Dodds et. al. Phys. Rev. A 100, 032320 \(2019\)](#)

*marked state of $|0\rangle$ w.l.o.g.

What is required? (last slide on aside) *

Need:

- ▶ Nearly all to all interactions, but only two body $\sigma_i^z \sigma_j^z$, don't need any higher order
- ▶ Single body σ_i^z and σ_i^x terms
- ▶ ... and that is it, nothing else required, we propose Rydbergs, but maybe one of you has a different idea...



Perturbatively recover simple search dynamics

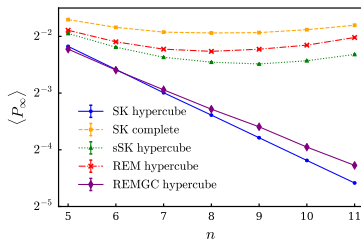
*Dodds et. al. Phys. Rev. A 100, 032320 (2019)

(Why) is the effect of correlations interesting?

If the bitstring energies are uncorrelated no *classical* algorithm could do better than random guessing, why?

Energy of one bitstring tells nothing about energy of neighbours

Complementary to the search-like mechanism usually attributed to quantum algorithms



Not just a 'one off' difference between spin glasses and REM, but more general between correlated and uncorrelated energies

Beyond simple quantum walks

Study of quantum walks on spin glasses fruitful for understanding computational mechanisms, but scaling is not cutting edge

How do we build better algorithms on top of this result?

1. Add a (rapid) quench to dissipate some energy
 - ▶ Need theory which goes beyond adiabatic and works for rapid quenches
2. Use as a hybrid subroutine along with classical computation
 - ▶ Needs to be coherent and ideally compatible with the theory from point 1

Rapid quenches?

The energy conservation argument given previously can be extended to any monotonic (closed system) quench

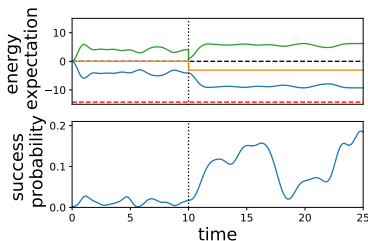
$$H(t) = A(t) H_d + B(t) H_{\text{problem}} \quad \frac{A(t)}{B(t)} \geq \frac{A(t + \delta t)}{B(t + \delta t)} \forall t$$

Sketch of proof:

1. Trotterize time evolution: $A(t) \rightarrow A(t + \delta t)$ and $B(t) \rightarrow B(t + \delta t)$ and apply $|\psi(t + \delta t)\rangle = \exp(-iH(t)\delta t)|\psi(t)\rangle$ in separate steps
2. Rescale time so that Hamiltonian always resembles quantum walk $H_{\text{eff}}(\gamma(t)) = \gamma(t) H_d + H_{\text{problem}}$
3. In rescaled version $\gamma(t) \geq \gamma(t + \delta t) \therefore \langle H_{\text{eff}}(\gamma(t)) \rangle_{\psi(t)} - \gamma(t) n \geq \langle H_{\text{eff}}(\gamma(t + \delta t)) \rangle_{\psi(t)} - \gamma(t + \delta t) n$
4. Because $\langle H_{\text{eff}}(\gamma(t)) \rangle_{\psi(t)} \geq -\gamma(t) n \forall t$, $\langle H_{\text{problem}} \rangle_{\psi(t)} \leq 0 \forall t$

Intuitive example: two stage quantum walk

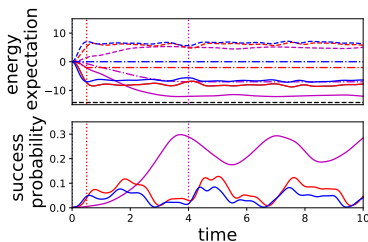
Perform a quantum walk at γ_1 , and then use result as an input state for a second walk at $\gamma_2 < \gamma_1$



- ▶ Energy expectations: **Green** = $\gamma_{1,2} \langle H_d \rangle$; **Blue** = $\langle H_{\text{problem}} \rangle$; **Gold** = $\gamma_{1,2} \langle H_d \rangle + \langle H_{\text{problem}} \rangle$
- ▶ Total energy conserved except for at dashed line where γ decreases
- ▶ Non-instantaneous quench effectively infinite stage quantum walk

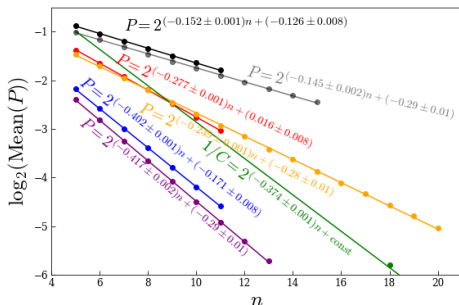
Pre-annealed quantum walk, single spin glass example

Perform an anneal before the walk to dissipate energy



- ▶ Vertical dashed line is end of pre-anneal
- ▶ Longer pre-anneal lowers $\langle H_{\text{problem}} \rangle$ (solid lines top plot) and raises success probability
- ▶ How does this affect scaling?
- ▶ Stop in paramagnetic regime and avoid exponentially small gaps in spin glass

Scaling boost from pre-annealing



- ▶ Blue and Magenta quantum walk (two different ways of choosing γ)
- ▶ Red and Gold Pre-annealed walks with γ values from regular QW
- ▶ Black and Gray Pre-annealed quantum walk with more optimal γ
- ▶ Green Effective scaling for classical branch-and-bound (for comparison)

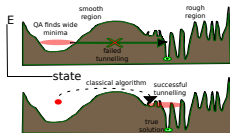
Pre-annealed quantum walk beats classical state of the art

- ▶ Thanks to Zoe Burtrand (summer project student at Durham) for optimal branch-and-bound (BnB) implementation
- ▶ Scaling exponent less than half of state-of-the-art classical
- ▶ Comparable to quantum branch-and-bound scaling exponent found in [arXiv:1906.10375](https://arxiv.org/abs/1906.10375) ours: 0.145, theirs 0.186

However...

- ▶ Our techniques are not hybrid like the quantum BnB (i.e. do not use classical tricks on top of quantum)
- ▶ Room for improvement as a subroutine in hybrid quantum classical? (maybe even combining with quantum BnB)

Hybrid subroutines in Continuous time: a review



Known techniques:

Reverse annealing [NC 2017 New J. Phys. 19 023024](#) as implemented on D-Wave devices

Relies on dissipation, not suitable for coherent algorithms

'Mexican hat' schedule [Perdomo-Ortiz et. al. Quantum Inf Process \(2011\) 10: 33. doi:10.1007/s11128-010-0168-z*](#)

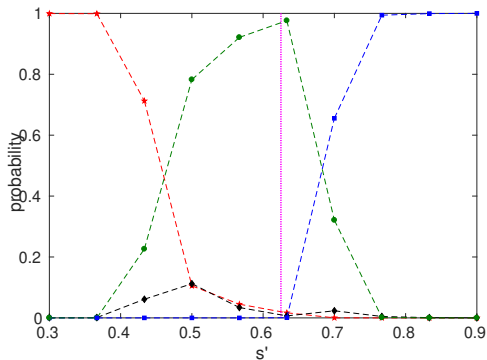
Involves three separate Hamiltonians, not compatible with rapid sweep proof

Biased driver Hamiltonian [arXiv:1906.02289](#) and [Chinese Physics Letters, 30 1 010302](#)

Compatible with proof, and can be used with quantum walk: this is the technique we want to focus on

*sometimes also called reverse annealing

Experimental biased search on a D-Wave device



- ▶ Level crossing, true GS and false minima at magenta line
- ▶ Anneal at maximum allowed rate, wait time (τ) of $20\mu S$
- ▶ Frozen in starting state for small s' , find true minimum at moderate s' , trapped for large s'

I can explain details of this experiment if people are interested

(more) Experimental results with dissipative reverse annealing

1. Start from one ground state to find other ground states ([D-Wave whitpaper 14-1018A-A](#))
 - ▶ [Finding other GS 150x more likely than forward](#)
2. Search locally around classical solution ([arXiv:1810.08584](#))
 - ▶ Start from greedy search solution
 - ▶ [Speedup of 100x over forward annealing](#)
3. Iterative search ([arXiv:1808.08721](#))
 - ▶ Iteratively increase search range until new solution found
 - ▶ [Forward annealing could not solve any, reverse solved most](#)
4. Quantum simulation ([Nature 560 456–460 \(2018\)](#))
 - ▶ Seed next call with result from previous
 - ▶ [Seeding with previous state makes simulation possible](#)
5. Monte Carlo and Genetic like algorithms
 - ▶ Quantum assisted genetic algorithm QAGA ([arXiv:1907.00707](#))
 - ▶ [Finds global optima quickly where traditional QA struggles](#)
 - ▶ Theoretical discussion ([NJP 19, 2, 023024 \(2017\)](#) and [arXiv:1609.05875](#))

Biased driver Hamiltonian*

Define driver Hamiltonian using fields which are not (completely) transverse $H_d = \sum_{i=1}^n -\cos(\theta)\sigma_i^x - g_i \sin(\theta)\sigma_i^z$

- ▶ Start in ground state of H_d :

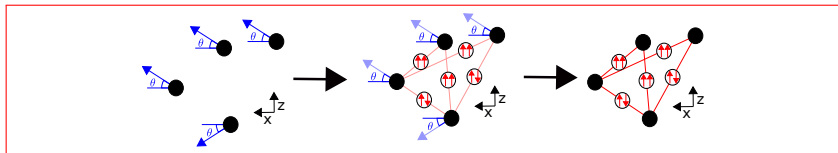
$$|\psi(t=0)\rangle = \bigotimes_{i=1}^n \frac{1}{\sqrt{2+2g_i \cos(\theta)}} [(1 + g_i \cos(\theta))|0\rangle + \sin(\theta)|1\rangle]$$

- ▶ Starting state biased toward classical bitstring g , $g_i \in \{-1, 1\}$
- ▶ Closed system with monotonic sweep (including QW), time evolution improves the guess (on average):

$$\langle H_{\text{problem}} \rangle_{\psi(t)} \leq \langle H_{\text{problem}} \rangle_{\psi(0)}$$

- ▶ Ground state is optimal solution so adiabatic theorem holds and dissipation can assist as well

Can use AQC, QW and QA mechanisms simultaneously

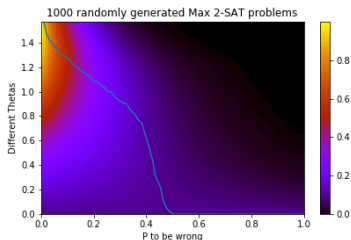


*with Laur Nita, Jie Chen, Adam Callison, Viv Kendon and Matthew Walsh.

Note related work: [arXiv:1906.02289](https://arxiv.org/abs/1906.02289) and [Chinese Physics Letters, 30 1 010302](https://doi.org/10.1016/j.cpl.2019.01.030)

Quantum walk with biased driver: proof-of-concept

- ▶ Consider a guess where each bit has an independent probability P of being wrong
- ▶ How good does the guess need to be before biasing (parametrized by θ , $\theta = 0$, no bias) improves the solution?



- ▶ Colour axis is success probability, line is optimal, result for eight qubit max-2-sat,
- ▶ hybrid techniques become useful right around 50% success probability, becomes significant around 45%
- ▶ Preliminary work by Laur Nita (PhD student)

Take home messages

Algorithms with exponentially low success probability in a single run

- ▶ Unless $P = NP$ all algorithms will have exponentially low success, exponential single run time, or both
- ▶ Only need to be coherent *for single run* → much less demanding for hardware (lower precision needed as well)
- ▶ Less psychologically satisfying, but no other real drawback

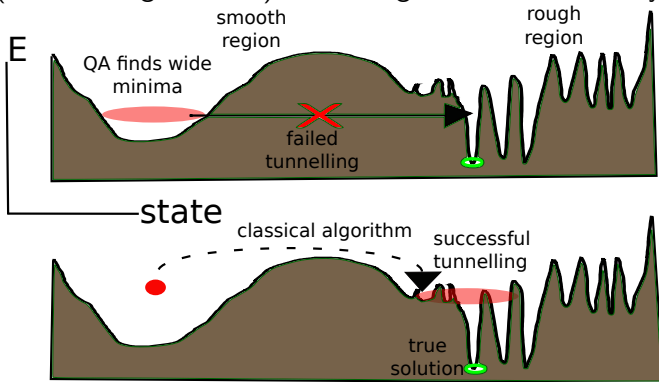
Quantum walks on spin glasses

- ▶ Correlations in energy landscape play a role, allow better than \sqrt{N} runtime
- ▶ Behaves differently from simple search, less demanding for control precision (**SS not good model for all computation!**)
- ▶ Pre-annealing → performance competitive with state of art
- ▶ Working on hybrid subroutines

Bonus story: more on the reverse annealing experiment

What do all reverse annealing algorithms have in common?

All rely on the concept of **local search**, more likely to find solutions close (in Hamming distance) to starting state, than far away



Important to show experimentally that reverse annealing = local search

How to test this (proof-of-principle version)

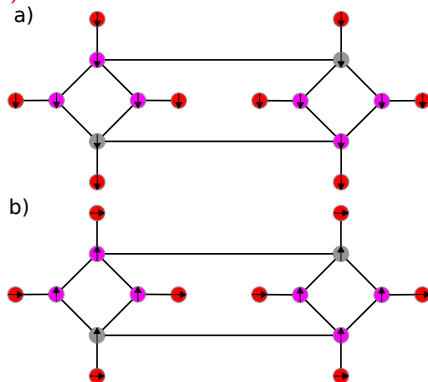
Need to construct a problem Hamiltonian with the following properties:

1. Wide false energy minimum which 'tricks' traditional quantum annealing algorithm
 2. Relatively narrow true minimum energy
 3. Local minimum near true minimum for start state
- ▶ For some value of the annealing parameter s_{cross} , fluctuations will lower the energy of the false minima below the true minima
 - ▶ Local search demonstrated by tunnelling to true minimum while $s < s_{\text{cross}}$



Constructing proof-of-principle Hamiltonian

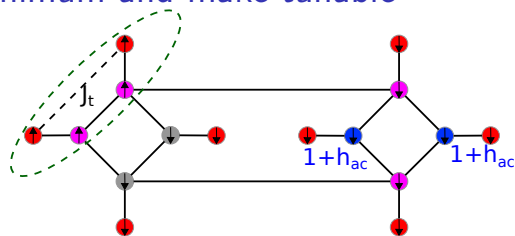
- ▶ Hamiltonians with features 1 and 2 are already known: free spin gadgets*
- ▶ Start with gadget from N. G. Dickson et. al. Nature Comm. 4, 1903 (2013)



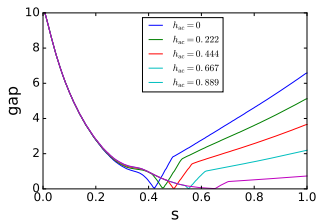
- ▶ a: unique ground state (red, $h=+1$ violet $h=-1$)
- ▶ b: 256-fold degenerate excited state \rightarrow false minimum

*See for instance: S. Boixo et. al. Nature Comm. 4, 3067 (2013)

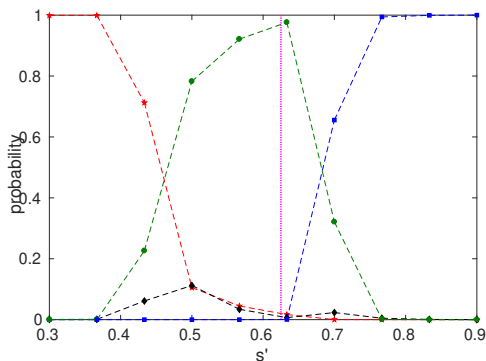
Add local minimum and make tunable



- ▶ Starting state shown by arrows, ground state except for circled spins flipped blue field is in - direction
- ▶ J_t controls barrier between start state and ground state.
- ▶ h_{ac} controls the value of s_{cross}



Experimental results *



- ▶ $J_t = 0.2$, $h_{ac} = 0.95$ level crossing between true GS and false minima at magenta line
- ▶ Anneal at maximum allowed rate, wait time (τ) of $20\mu S$
- ▶ Frozen in starting state for small s' , find true minimum at moderate s' , trapped for large s'

*For different proof-of-principle results, see: [D-Wave white paper on Reverse Quantum Annealing for Local Refinement of Solutions](#)

Future work

Have demonstrated the principles behind local search now to demonstrate it is a useful algorithmic tool

- ▶ Can we find better solutions starting from the best traditional QA can find (simple and immediate demonstration that reverse annealing is useful)
- ▶ Use problems with known solutions to test range over which reverse annealing can search
- ▶ Test sampling applications: balancing tradeoff between memory and search range

