# Modenizing Quantum Annealing using Local Search

EMiT 2017 Manchester

Based on: NJP 19, 2, 023024 (2017) and ar$\chi$iv:1609.05875

Nicholas Chancellor

Dec. 13, 2017

# Outline

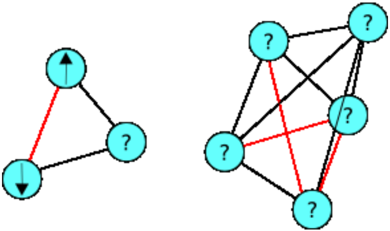
1. Energy Computing and the Ising Model
2. Quantum Annealing and Simulated Annealing
   - Better Classical Algorithms: Parallel Tempering and Population Annealing
   - Hybrid Computing: Gaining the Advantages of Advanced Algorithms
   - Numerical example
3. One slide aside: problem misspecification
4. Inference primitive formalism
   - Simple examples, traditional Quantum annealing and repeated local search
   - More complicated Parallel Tempering and Population Annealing algorithms
   - Gentic algorithms
5. Conclusion

# Problem Statement: Ising Spin Glass Hamiltonian

$$H_{ISG} = \sum_i h_i \sigma_i^z + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

Goal is to find ground/low energy states

- 'Universal' in the sense that any classical Hamiltonian can be mapped to it De las Cuevas, Cubitt Science 351 6278
- Thermal/quantum distributions also useful for inference and machine learning tasks ex. Amin et. al. arXiv:1601.02036, Chancellor et. al. Scientific Reports 6, 22318 ...

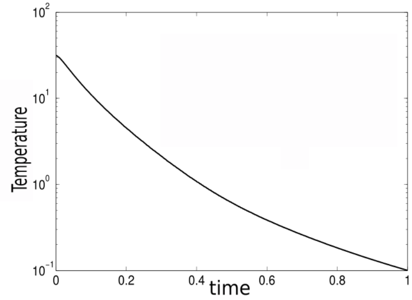


# Simulated Annealing (classical)

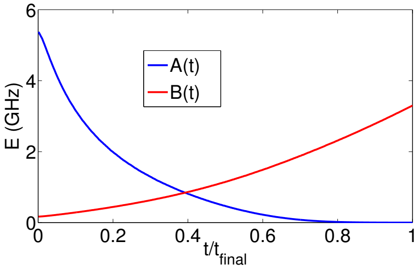
Updates drive toward thermal distribution with temperature $T$ if they obey detailed balance

$$P(S(1) \to S(2)) = \exp(\frac{(E(1) - E(2))}{T}) P(S(2) \to S(1))$$

Start at high $T$ and lower over time



# Quantum Annealing (QA)



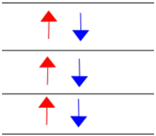

Add non-commuting transverse field terms

$$H(s) = -A(s) \sum_i \sigma_i^x + B(s) H_{ISG}$$

start at $\frac{A(s=0)}{B(s=0)} \gg 1$, go to $\frac{B(s=1)}{A(s=1)} \gg 1$
Quantum fluctuations + low temperature bath cause tunneling toward low energy states

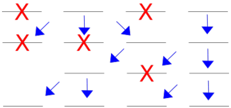
# Beyond Simulated Annealing (classical)

Parallel Tempering



- Multiple replicas at different temperatures
- Swap replicas by rules which obey detailed balance
- $P_{swap}(i,j) = \min\left[1, \exp\left(\left(\frac{1}{T(i)} - \frac{1}{T(j)}\right)(E_i - E_j)\right)\right]$

Population Annealing



- Anneal multiple replicas
- Probabilistically remove poorly performing replicas and copy those which perform well
- Rules preserve average population and obey detailed balance
- $\bar{N}(E) = \frac{1}{Q}\exp\left(\left(\frac{1}{T_{old}} - \frac{1}{T_{new}}\right)E\right)$

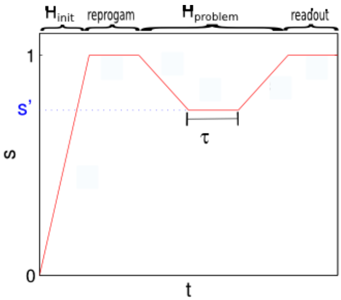
Can hybrid strategies combine these with calls to an annealer?

Can these strategies be used directly by a quantum annealer?

# Difficulties in building new annealer strategies

- No cloning theorem → cannot copy quantum states
- Measurements (ex. energy) disturb state of system and likely experimentally difficult
- Usual QA is global search, no way of inserting information

Solution → use annealer subroutine which starts and ends at $s = 1$ (recall $\frac{B(s=1)}{A(s=1)} \gg 1$) with programmed initial state [1]




[1] for an alternative closed system approach, see: A. Perdomo-Ortiz, et. al. Quant. Inf. Proc.10(1):33–52, (2011). See also T. Graß and M. Lewenstein Phys. Rev. A **95**, 052309 (2017).

# Doing this experimentally

- Only based on altering the classical control protocol of devices, does not require changes to the qubits themselves
- This feature will be included in quantum annealing devices manufactured by D-Wave Systems Inc. [2]

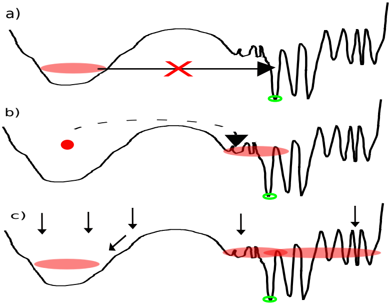
[2] Reverse annealing logo created by D-Wave Systems Inc. used with permission.

# Hybrid computing using local search of solution space

Potential Strategies

1. Quantum and classical algorithms used together
   - Classical input and output means that annealer can be used alongside any classical algorithm
2. Multiple local quantum searches controlled by classical algorithm
   - Analogues to parallel tempering and population annealing which use annealer only
   - Will return to this later

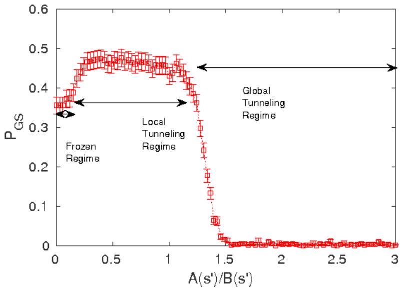
# Cartoon example: energy landscape with rough and smooth features




a) QA gets stuck in broad local minima and cannot tunnel to correct minima
b) Classical algorithms can easily explore the broad features, while the annealer can explore the rough ones
c) Even random initialization can improve solution probabilities, may hit rough region by chance

# Numerical proof-of-principle

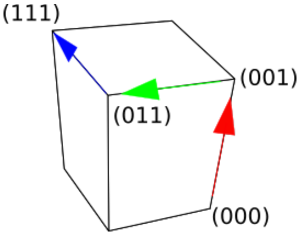
- We can construct a 16 qubit Hamiltonian with energy landscapes like the one shown on the previous slide
- Start from a random state as depicted in (c)
- Reverse anneal to different $s'$ values




- $\frac{A(s')}{B(s')} \approx 0$ the dynamics are effectively frozen
- moderate $\frac{A(s')}{B(s')}$, search locally $\rightarrow$ improve solution
- large $\frac{A(s')}{B(s')}$ get trapped in a false minima $\rightarrow$ performs poorly

# One slide aside: robustness against Problem mis-specification

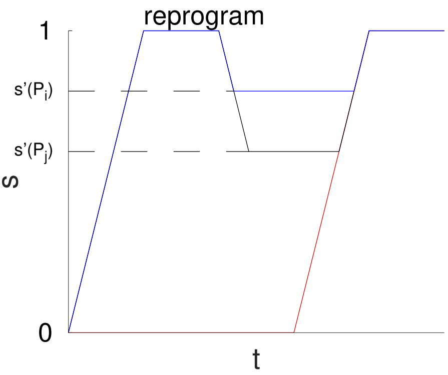
- Error in each energy proportional to $\sqrt{N_{qubit}}$
- Only energy differences within search matter
- Energy difference proportional to square root of Hamming distance
- ∴ relevant error proportional to square root of search range not $\sqrt{N_{qubit}}$ [3]




[3] Up to details about shape of the explored subspace, see NJP 19, 2, 023024 (2017)

# Including uncertainty by annealing qubits differently

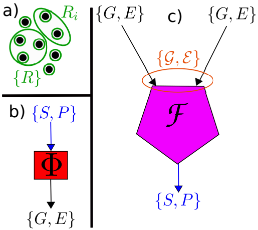
What if we are more sure about some parts of our guess then others? $\rightarrow$ anneal different qubits back to different points



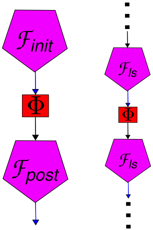

An extreme version of this, which excluded qubits where a value was expected with high certainty has already been done H. Karimi and G. Rosenberg Quantum Inf. Proc. 16(7):166 (2017) and H. Karimi and G. Rosenberg Phys. Rev. E, 96:043312

# Representing this graphically: Inference Primitive Formalism

- Represent quantum annealing call as an inference primitive $\Phi$, takes state guess $S \in \{-1, 1\}$ and uncertianty values $P \in [0, 0.5]$, outputs list of states $G$ and energies $E$
- Processing function $\mathcal{F}$ represents classical processing $\rightarrow$ takes any number (including zero) of annealer outputs (found states $G$ and energies $E$) and finds new guess $S$ and uncertainty values $P$
- Easily generalized to multi-body drivers representing uncertainty on clusters of qubits



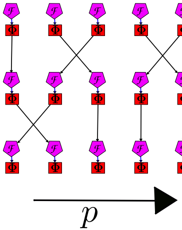
# Basic Examples: traditional QA, and repeated local search in this formalism

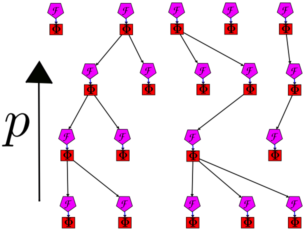



- Traditional QA (left) represented by initialization processing function which takes no inputs and gives complete uncertainty ($P_i = 0.5 \forall i$) on all qubits, followed by post processing function
- Repeated local search (right) from running annealer many times and using the output as an input to the next processing function

# More advanced algorithms: Parallel tempering and Population annealing analogues

- Processing function $\mathcal{F}$ returns lowest energy state as guess and gives all qubits the same uncertainty $P_i = p \forall i$
- Assign effective temperature $T$ to each $p$ value and either:
  1. exchange using Parallel tempering rules (left)
     $P_{swap}(i,j) = \min\left[1, \exp\left(\left(\frac{1}{T(i)} - \frac{1}{T(j)}\right)(E_i - E_j)\right)\right]$
  2. kill or replicate states using population annealing rules (right)
     $\bar{N}(E) = \frac{1}{Q} \exp\left(\left(\frac{1}{T_{old}} - \frac{1}{T_{new}}\right) E\right)$



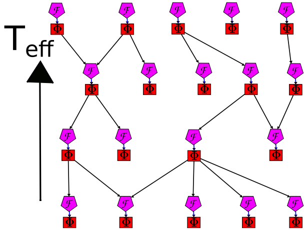
# Even more advanced algorithms: Genetic algorithms

- A processing function which takes more than one input is a ‘breeding’ step of a genetic algorithm
- For instance could be thermally reweighted sum[4] ( u indicates sum over unique states found)

$S_i = \mathrm{sgn}(\sum_{j=1}^{N_u} G_j^{(u)} \exp(-\frac{E_j^{(u)}}{T_{eff}})),$

$P_i = \frac{1}{Z}(\sum_{j=1}^{N_u} \delta_{G_j^{(u)},-S_i} \exp(-\frac{E_j^{(u)}}{T_{eff}}))$

- Could be used to add crossbreeding to Population annealing analogue, as shown below




[4]see: arχiv:1609.05875 for details

# Conclusions

- Classical controls can be used to make quantum annealers perform subroutines in hybrid quantum/classical algorithms
- D-Wave Systems Inc. are already implementing these controls on their devices
- Annealer call can be represented in the inference primitive formalism
- Many algorithmic possibilities, including genetic algorithms, this work has barely scratched the surface

# Acknowledgements


- Thanks to Viv Kendon for multiple critical readings of the paper
- Work supported by EPSRC
- You $\rightarrow$ thanks for listening


Please read the full papers: NJP 19, 2, 023024 (2017) and ar$\chi$iv:1609.05875